2011 Mathematical Contest in Modeling (MCM) Control Sheet

Advisor Name: Zuguo He

Department: Electronic Engineering

Institution: Beijing Univ. of Posts & Telecomm.

Address: BOX 91 Xi Tu Cheng ROAD 10 HaidianDISTRICTBeijing University of Posts and Telecomm.
Beijing, Beijing 100876

Phone: +86-13041127820

Fax: +86-10-62282931

Email: hezuguo@bupt.edu.cn

Home Phone: +86-13041127820

Your team's control number is:
10906
(Place this control number on all pages of your solution paper and on any support material.)

Problem Chosen:
B

The names of the team members will appear on your team's certificate **exactly** as they appear on this page, including all capitalization and punctuation, if any. Gender data is optional and will be used for statistical purposes only; it will not appear on the certificate.

Team Member	Gender
XiaohuiLuo	F
Junlong Chen	M
Ziqiang Liu	M

Each team member must sign the statement below:
(Failure to obtain signatures from each team member will result in disqualification of the entire team.)

Each of us hereby testifies that our team abided by all of the contest's rules and did not consult with anyone who was not on this team in developing the enclosed solution paper.

luoxiaohui@bupt.edu.cn

Mailing Address and Signature of Xiaohui Luo

cjllife@yahoo.cn

Mailing Address and Signature of Junlong Chen

byliuziq@163.com

Mailing Address and Signature of Ziqiang Liu

This signed original must be stapled to the top of one copy of your team's solution paper.

For office use only	Team Control Number	For office use only
T1 ________________	**10906**	F1 ________________
T2 ________________		F2 ________________
T3 ________________	Problem Chosen	F3 ________________
T4 ________________	**B**	F4 ________________

2011 Mathematical Contest in Modeling (MCM) Summary Sheet
(Attach a copy of this page to each copy of your solution paper.)

Type a summary of your results on this page. Do not include the name of your school, advisor, or team members on this page.

Summary

Repeaters play a promising role of constructing a communication system in a limited area. In this paper, we present a discrete integer model to find the minimum number of repeaters and optimize the placement method.

First, we value the performance of the cellular network against the other polygon network, proving that the cellular network is the best. Moreover we determine some crucial parameters and conclude several principles of communicating process by referring to an amount of information from professional books and the Internet. Armed with the knowledge, we make out the method of feasible communication system.

Then, to make the best of both coverage and channel capacity, we design a discrete integer model base on as uniform distribution's population to determine the minimum number of repeaters and the corresponding placement method.

Combined with the principles of communication process and optimization model for placement, we offer three solutions respectively to the problems.

- In task one, communications happen in the flat area with a low population density, so the channel resource is enough. We find a relatively easy optimal placement of repeaters.
- In task two, the population density increases so that the channel resources become limited. Therefore, we change the size of the cell and add the number of repeaters placed in one cell to meet the users' demand according to the discrete integer model's results.
- In task three, we average the effect of mountainous terrain to the whole area, so solution can be provided by borrowing the models in task one and task two.

To test the practicability of our models in real-life scenarios, we use the computer to stimulate the situation where the users are randomly distributed, in which case, by bringing in two reliable criterion, we test the service's quality our communicating models could provide.

Finally, we measure the sensitivity of our models and analyze the strength and weakness. Moreover, we provide some envision into the future work about this system.

Key Word: Repeater CTCSS VHF Cellular Network

Optimal Placement of Repeaters Based on Cellular Network

Contents

1. Introduction

VHF is commonly used in wireless communication. However, since its propagation characteristics of radio waves are very vulnerable to blockages and its power is usually low, the distance of corresponding communication meets limitations. To overcome this limitation, repeater, which picks up weak signals, amplify them, retransmit them on a different frequency, is brought into use. But there are still interference problems needed to be solved. One solution is geographical separation, and another is using CTCSS technique to identify adjacent repeaters apart.

In this setting, our goal is to develop an optimization framework which computes the minimum number of repeaters and their corresponding placements and PL tone assignments in the network.

Due to the enormousness of designs by researchers in real-life scenario, we borrow the cellular mobile communication system to expend our models. In our designing, there are some crucial parameters:

- the radius of operating range of repeater in flat area scenario
- the radius of operating range of repeater in mountainous scenario
- the maximum number of users that a repeater can serve at the same time
- the size of each cell in cellular mobile communication system
- the method of placement of repeaters
- the method of placement of PL tones

Then, to accomplish this, we break down our task as follows:

- Validate the cellular network as the optimal model to build the communication system against a mathematical analysis;
- Calculate the respective radius of operating range of repeater in flat area scenario and in mountainous scenario;
- Find an optimal size of cells and a method of placement of repeaters to achieve the total area radio coverage to meet the demand of facilitating a simultaneous communication for a given number of people;
- Distribute the PL tones to every repeater to eliminate the interference between the adjacent repeaters;
- Use a random distribution of population of simulator to analyze the performance of our models;
- Provide an overview of the advantages and disadvantages of our approach and give directions for future work.

2. General Assumptions

- All repeaters have the same constant parameters.
- The population distribution in this area conforms to uniform distribution.
- A area of 40 miles radius refers to a standard circular region.
- At a certain time, one channel of the repeater can only provide service to one pair of users.
- The receiver and transmitter in the communication are both located in the 40 miles radius's area we studied.
- The propagation delay in the communication process is neglected in our system.

3.Terminology

- **VHF**[1]: a very high frequency which ranges from 30MHz to 300MHz.
- **Line-of-sight**[2]: electro-magnetic waves travelling in a straight line.
- **Repeater**[3]: an electronic device that receives a signal and retransmitsit at a higher level and/or higher power, or onto the other side of an obstruction, so that the signal can cover longer distances.
- **Continuous tone-coded squelch system(CTCSS)** [4]: a circuit that is used to reduce the annoyance of listening to other users on a shared two-way radio communications channel. It is sometimes called tone squelch. Where more than one user group is on the same channel (called co-channel users), CTCSS filters out other users if they are using a different CTCSS tone or no CTCSS.
- **Cellular mobile communication**[5]: a system uses a large number of low-power wireless transmitters to create cells—the basic geographic service area of a wireless communications system.
- **AMPS**[6]: an analog mobile phone system standard developed by Bell Labs, and officially introduced in the America in 1983, Israel in 1986, and Australia in 1987.

4.Problem Background

The VHF radio spectrum which involvesline-of–sight transmission and reception is ideal for short-distance terrestrial communication, however, in the same time, it is confronted with the distance limitation for its low power and vulnerability of the blockage. Thus, using a repeater, which picks up weak signals, amplify them, and retransmit them on a different frequency, low-power users can communicate with one another in situations where direct user-to-user contact would not be possible. In real-life scenario, people employ the cellular mobile communication system to place the repeaters to implement a total area radio coverage.

A cellular mobile communications system uses a large amount of wireless transmitters to create cells---the basic geographic service area of a wireless communications system. And the cells are sized according to the subscriber density and demand within a particular region. Each mobile talks to the cell site by using a separate, temporary radio channel. The cell site talks to many mobiles at once, using one channel per mobile. Channels use a pair of frequencies for communication---one frequency, the uplink, for transmitting from the cell site, and one frequency, the downlink, for the cell site to receive calls from the cell site[5], which is demonstrated clearly in **Figure 1**.

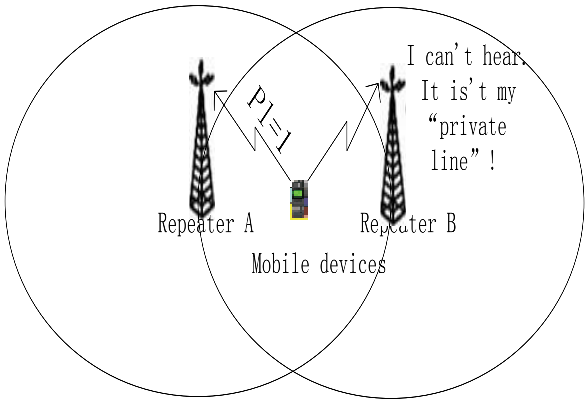


Figure 1 communication process

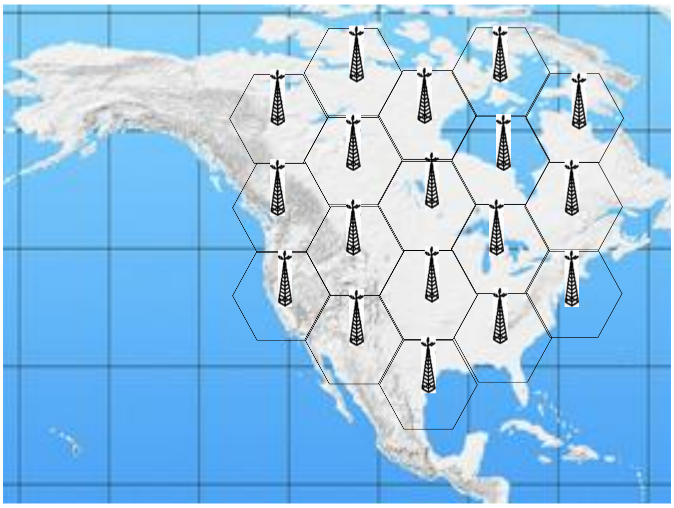


Figure 2 cellular network

Figure 2 shows a mobile telephone system using a cellular architecture. However, mobile communication will meet a big interference problem when the adjacent areas use the same channel. Traditionally, different frequencies should be used between the adjacent areas, which lead to a low utilization ratio of frequency. Fortunately, the advent of CTCSS technique casts a light on the solution to this problem. By using a unique PL tone to identify each repeater, two nearby repeaters can share the same frequency pair; so more repeaters can be accommodated in a particular area.

Acquiring the knowledge stated above, our goal is to construct an optimal communication system based on the cellular network that minimizes the number of repeaters placed in it and meets the demand of communication of users.

5. The Models

5.1 Introduce

In this section, firstly, we value the cellular network against some other polygon network to prove that the cellular network is the best frame for our work. Secondly, after referring to an amount of information, we determine some crucial parameters and make out the principle for communication process. Thirdly, to minimize the number of repeaters, we build a discrete integer model with a uniform distribution to design an optimal placement method. Then, with all the data and methods concluded formerly we accomplish the three tasks. Finally, we construct a discrete integer model with a random distribution to validate our models.

5.2 Value the Cellular Network

Since the Bell Labs introduce the concept of cellular network in 1970s, the mobile communication has officially come into commercialization. The mobile communication system has largely improved the system capacity by using the cellular network to achieve spatial multiplexing.

And it is the true solution to the conflict between big capacities and limited wireless frequency resources. So far the cellular mobile communication system has worked successfully in the first generation of analog communication system, and still deployed by the second and third generation of analog communication system.

The hexagonal structure of cellular network is the best form of cell shape. It is common sense that the radiation area of omnidirectional antenna in the desired coverage area should be the circle based on the center of the antenna radiation source. In order to achieve seamless coverage, the circles produced by each antenna radiation source are bound to overlap, which, in terms of communication, means interference. To determine which polygon can minimize the area of overlap, we choose some kinds of polygons to calculate the overlap and compare with each other, which is demonstrated in**Table 1.**

Table 1 Effect comparison between different polygons system

Shape	Regulartriangle	Square	Regularhexagon
Distance	r	$\sqrt{2}$r	$\sqrt{3}$r
area	1.3r^2	2.0r^2	2.6r^2
overlap	1.2πr^2	0.73πr^2	0.35πr^2

According to the above data, we can clearly conclude that the hexagonal holds the minimized overlap in situation that all polygons can seamlessly cover the same size of service area. So the hexagonal structure of cellular network is much preferred by people in reality. In our paper, we adopt it as the basis form of our communication system.

5.3 Crucial Parameters in Our Models

In order to make the communication system work successfully and meet the demand of users, there are some parameters playing an important role in the communicating process. To the best of our knowledge, we should determine:

- $\mathbf{R_{UtoR}}$(**uplink distance**):the maximum distance between a user and a repeater when the user is sending signals to the repeater. It determines the maximum size of every cell in cellular network.
- $\mathbf{R_{RtoU}}$(**downlink distance**)**:** the maximum distance between a repeater and a user when the repeaterissending signals to the user.
- $\mathbf{N_{ch}}$**:** the maximumnumber of channels that a repeater can hold. It constrains the actual size of every cell in cellular network according to the density of population.

After browsing the Internet[7], we find a practical formula to calculate these distances.

$$LM(dB) = 88.1 + 20\lg F - 20\lg h_1 h_2 + 40\lg d \tag{1.1}$$

In the formula:

F----the working frequency in communication

h_1---the height of transmitter's antenna

h_2---the height of receiver's antenna

d-----the distance between the transmitter and the receiver

When calculating the $\mathbf{R_{UtoR}}$, the transmitter is the user, and the receiver is the repeater. Their corresponding parameters are showed in **Table 2.**

Table 2 Parameters

Repeater and Erection parameters		Interphone parameters	
Transmitter power	20w(43dbm)	Transmitter power	4w(36dbm)
ReceiveSensitivity	-116dbm	Receiver Sensitivity	-116dbm
Coaxial cable loss	2db	Wireless Gain	0dbi
Wireless Gain	9.8dbi	Height	1.5m
Height	30m		

When calculating the $LM(dB)$, we can use the formula:

$$LM(dB) = Pt + PA - (PA + CL + RR) \tag{1.2}$$

In the formula:
Pt----the working frequency of transmitter
PA---the wireless gain of transmitter
RA---the wireless gain of reciever
CL----the coaxial cable loss
RR----the receiver sensitivity

According to the parameters in Table 2, we get LM =144.2(dB) through the formula (1.2). Then through the formula (1.1), we have the final results---R_{UtoR} in flat area is 8 miles. Referring to some information, we find that LM should decrease to 136.2(dB) because of the obstacle of hills. Therefore, R_{UtoR} in mountainous area is correspondingly 5 miles.

As to theN_{ch} , referring to *Radio Enthusiast*[8], the frequency channel for each user is 12.5 KHz. What's more, we assume that the bandwidth of a repeater is 600 KHz, because the interval between itsreceiving frequency and transmitting frequency is 600KHz. So we get the N_{ch} :

$$N_{ch} = \frac{600}{12.5} = 48$$

In conclusion, the final results are demonstrated in **Table 3**:

Table 3 Results

parameters	results
R_{UtoR} in flat area	8 miles
R_{UtoR} in mountainous area	5 miles
R_{RtoU}	100 miles
N_{ch}	48

5.4 Principles of Achieving Communication in the Cellular Network

According to the information from the question and some corresponding communication knowledge we referred, we get some crucial conclusion as following:

- **About the repeaters**

Since the transmitter frequency in a repeater is either 600 kHz above or 600 kHz below the receiver frequency, in order to avoid the interference between the two kinds of frequencies, it is easily concluded that the receiver frequency and the transmitter frequency couldn't overlap. In this case, repeaters can be classified into four groups from the 145 MHz to 148 MHz at an interval of 600 KHz. This is clearly showed in **Table 4**.

Table 4 Repeater Groups

Group	Receiver Frequency(MHz)	TransmitterFrequency(MHz)
A	145.0 ——— 145.6	145.6 ——— 146.1
B	145.6 ——— 146.1	146.1———146.7
C	146.1——— 146.7	146.7 ———147.3
D	146.7 ———147.3	147.3 ———147.8

- **About the PL tones**

From the classification of repeaters, we can see that there are two types of interferences:

One is the ***same frequency interference***, which happens when two repeaters from the same group have the overlap of uplink area, as showed in **Figure 3**

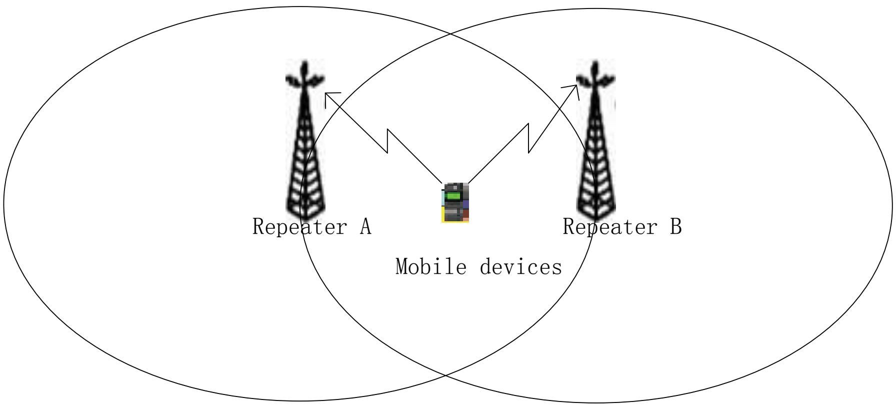


Figure 3: the same frequency interference

The other is the***second intermodulation interference***.

Then we use the PL tones to eliminate those two kinds of interference. For the same frequency interference, we distribute the PL tones to the repeaters from the same groups for their identity. For the second intermodulation interference, we attach the repeaters from the adjacent groups --- group A and group B, group B and group C, group C and group D---different PL tones to set them apart. Then we find out that:

$$S_i \le 54$$

S_i stands for a set number of the adjacent cells from any group $i \in (A, B, C, D)$

- **About the uplink distance and the downlink distance**

The uplink distance constrains the size of each cell in cellular network. From the Table 3, we can draw that the uplink distance is limited so that there must be enough number of repeaters to make every user available to attach the repeater.

The Table 3 tells us that the downlink distance is larger than 40 miles, which means that no matter where the user is, the signals sent by the repeater can be available to the user.

- **About the process of communicating**

There are two kinds of communication situation in our system---communicating in one cell and communicating between two cells. The **Figure 5** and **Figure 6** respectively show the two kinds of process.

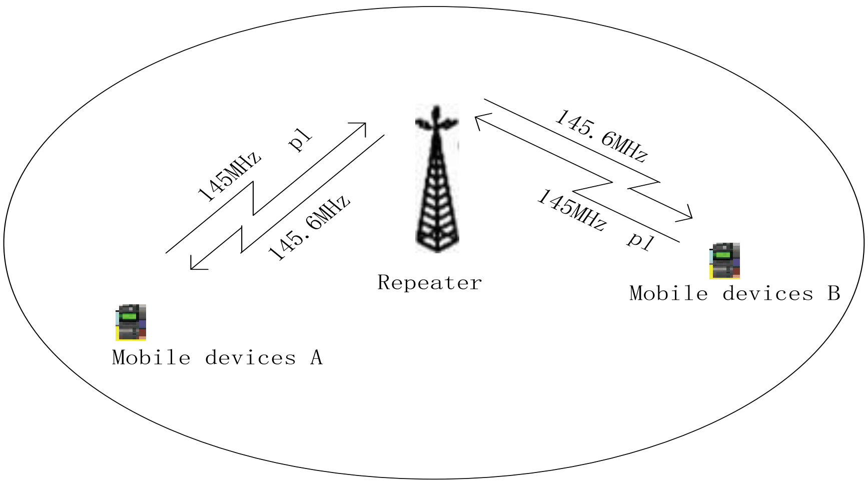


Figure 4 communicating in one cell

Figure 5 shows the process of communicating in one cell. The users will be distributed a pair of receiver frequency and transmitter frequency when communicating. And just one repeater serve for them.

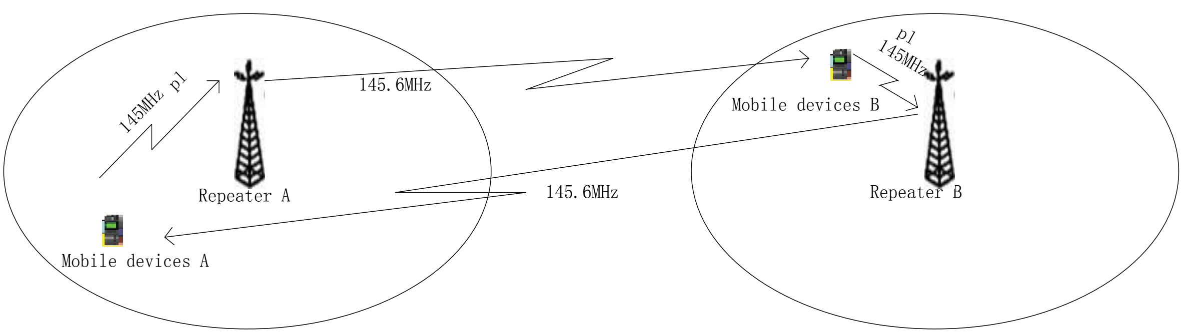


Figure 5 communicating between two cells

Figure 5 demonstrates the process of communicating between two cells. We can see that there are two repeaters serving for them, and the two repeaters must come from the same group, which means they can receive the same frequency, however, they needn't hold the same PL tones. The users will also be distributed a pair of receiver frequency and transmitter frequency when communicating.

5.5 A Discrete Integer Model to Optimize the Placement of Repeaters

5.5.1 Introduce the Model

Intuitively we know that the cellular network isn't equal to the circle. So in the edge the cellular network will not match the circle perfectly, which means this part will be promising to get improved.

To form the cellular network, inevitably we will be confronted with the problem whether or not the cell in the edge get included in the system. Therefore, we design a "Choice Rule" to deal with these situations.

"Choice Rule": For cells in the edge, if the center of a cell is inside the circle, the cell will be concluded in the network. If the center of a cell is outside the circle, the cell will be excluded from the network.

In **Figure 7**, we can see that if casually placing the cellular network, the cover is not perfect and the number of cells is not a minimum. To make up this defect, we build a discrete integer model to optimize the placement.

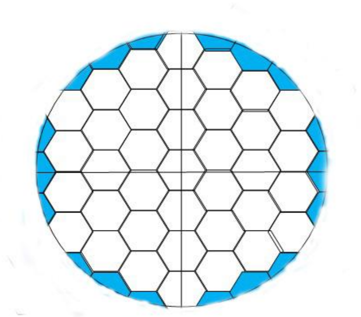


Figure 6

5.5.2 Symbols Definition

Symbols	Definition
R	The radius of the whole area
r	The length of the hexagon's side
S_{cir}	The area of the whole area
S_{hex}	The area of each cell
N_{ideal}	The ideal number of cells in cellular network
N_p	The practical number of cells in cellular network
P	The number of repeaters in each cell
M_{total}	The total number of people
M_{ideal}	The ideal number of people in each cell
M_p	The practical number of people in each cell

5.5.3 Model Details

Our goal is to find the optimal placement of repeaters to minimize the number of repeaters used. In terms of that, we describe our model in mathematical terms as following:

$$Min \quad N_p \times P$$
$$s.t \qquad P = \left\lfloor \frac{M_p}{C} \right\rfloor \quad r < r_{\max}$$

,

Using the geometric relations, given the situation that the people are uniformly distributed, we calculate M_p as following:

$$S_{hex} = \frac{\sqrt{3}}{4} r^2 \times 6 = \frac{3\sqrt{3}}{2} r^2$$
$$\mathrm{M}_p = \left\lceil \frac{\mathrm{M}_{total} \times S_{hex}}{S_{cir}} \right\rceil$$

The main of our work is to calculate the variable N_p. And to get N_p, we design two strategies. Now we will demonstrate them in details.

- Strategy One

In strategy one, the cellular network expands based on the center of the circle. We can see the macroscopic view in **Figure 7** and microscopic view in **Figure 8**.

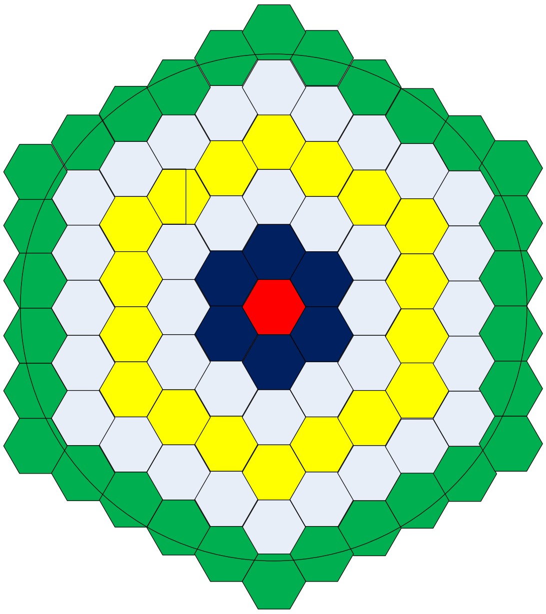


Figure 7: macroscopic view

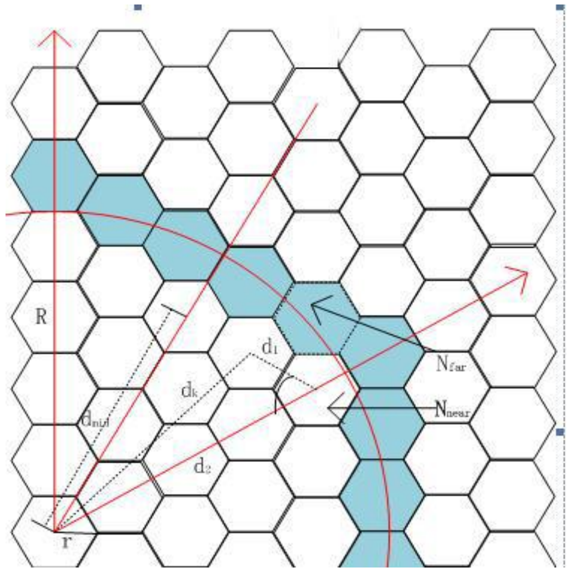


Figure 8: microscopic view

1) As showed in Figure 8, we call the cells that are nearest to the circle but are totally concluded in the circle as "near layer", and correspondingly, we mark the cells that are nearest to the circle but are not totally concluded in the circle as "far layer".
   The distance of "near layer" to the center of circle is N_{near} :

$$N_{near} = \left\lceil \left(R - \frac{\sqrt{3}}{2} r \right) / \sqrt{3} r \right\rceil + 1$$

   Similarly,the distance of "far layer" to the center of circle is N_{far} :

$$d_{mid} = (N_{near} - 1) \times \sqrt{3} r \times \cos(\frac{\pi}{6}) = \frac{3r}{2}(N_{near} - 1)$$

$$N_{far} = N_{near} + \left\lfloor \frac{2(R - d_{mid})}{3r} \right\rfloor$$

2) Calculating the cells surrounded by "far layer" cells, we get the total number of cells is N:

$$N = 1 + 6 + 12 + 18 + \cdots + (N_{near} - 2) \times 6$$
$$= 1 + 3(N_{near} - 1)(N_{near} - 2)$$

3) The distance of cells in "near layer" and "far layer" to the center of circle is d_k .
   And $k \in [1, (n-1) * 6]$

$$d_1 = (k-1) \times \sqrt{3} r$$
$$d_2 = (n-1) \times \sqrt{3} r$$

$$d_k = \sqrt{d_1^2 + d_2^2 - 2 d_1 d_1 \cos(\frac{\pi}{3})} = \sqrt{3} r \sqrt{(n-1)^2 + (k-1)^2 - (k-1)(n-1)}$$

According to the "Choice Rule", if the $d_k \le R$, the corresponding cell will be concluded in the cellular network, vice verse.

Then we use N_{temp} to count the number of cells from "near layer" and "far layer" which are concluded in the network by the "Choice Rule"

4) Based on the data above, we get the N_p:

$$N_p = 1 + 3(N_{near} - 1)(N_{near} - 2) + N_{temp}$$

- Strategy Two

In strategy two, the cellular network does not expand based on the center of the circle. We can see the macroscopic view in **Figure 9** and microscopic view in **Figure 10**.

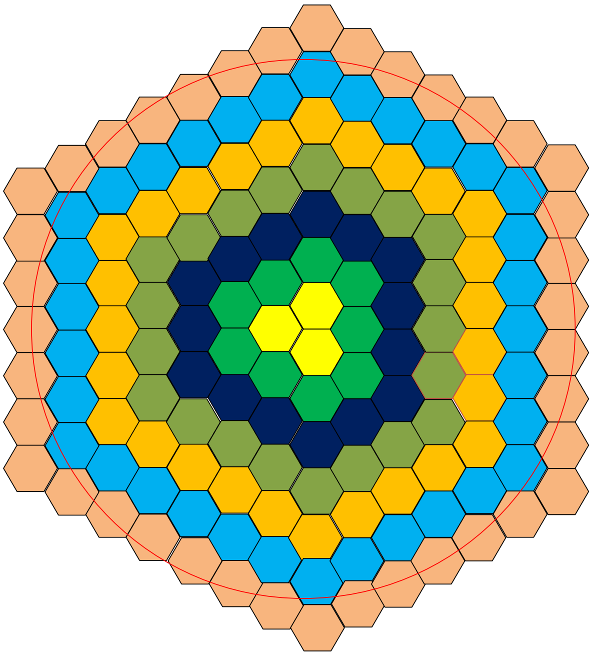


Figure 9: macroscopic view

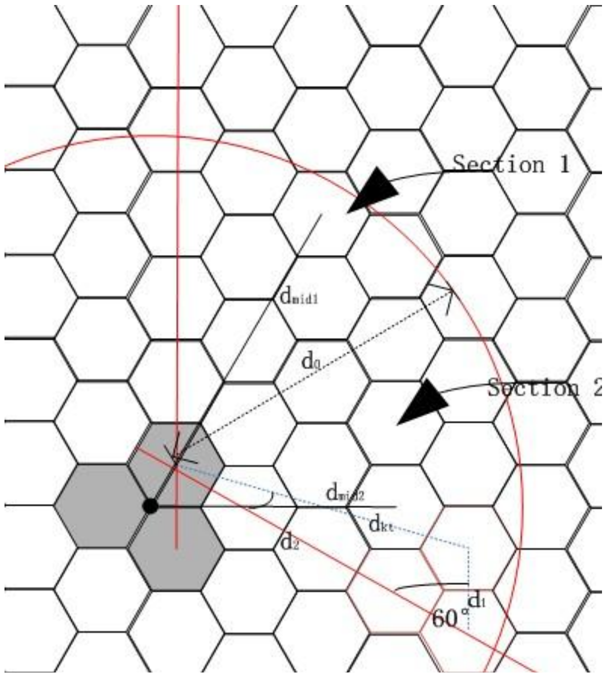


Figure 10: microscopic view

1) In this strategy, we have the same definition of "near layer".
According to the Pythagorean theorem, there are following relations:

$$d^2 + r^2 - 2dr\cos\theta = R^2$$

$$\Rightarrow d = r\cos\theta + \sqrt{R^2 - r^2\sin^2\theta}$$

$$d_0 = d\,|_{\theta=\frac{5\pi}{6}} = -\frac{\sqrt{3}}{2}r + \sqrt{R^2 - \frac{r^2}{4}}$$

Then we get N_{near}:

$$N_{near} = \left\lceil \frac{d_0}{\sqrt{3}r} \right\rceil$$

However, we have slightly different definition for "far layer". The "far layer" cells in section 1 showed in Figure 10 are called "far layer 1", and the "far layer" cells in section 2 showed in Figure 10 are called "far layer 2". Then, similarly, there are N_{far1}, N_{far2}.

$$d_{mid1}=(n_{near}-1)\times\sqrt{3}r\times\cos(\frac{\pi}{6})+r=\frac{3r}{2}(n_{near}-1)+r$$

$$\Delta N_1=\left\lfloor\left(R-d_{\mathrm{mi}d1}\right)/\frac{3r}{2}\right\rfloor$$

$$N_{far1}=N_{near1}+\Delta N_1$$

$$d_{mid1}=(n_{near}-1)\times\sqrt{3}r\times\cos(\frac{\pi}{6})+\frac{r}{2}=\frac{3r}{2}(n_{near}-1)+\frac{r}{2}$$

$$\Delta N_2=\left\lfloor\left(R-d_{\mathrm{mi}d2}\right)/\frac{3r}{2}\right\rfloor$$

$$N_{far2}=N_{near2}+\Delta N_2$$

2) Calculating the cells surrounded by the "far layer 1" and the "far layer 2" cells, we get the total number of cells is N:

$$N=1+6+12+18+\cdots+(N_{near}-2)\times 6$$
$$=1+3(N_{near}-1)(N_{near}-2)$$
$$=3(N_{near}-1)^2$$

3) In section 1 and section 2, the distance of cells in "near layer" and "far layer" to the center of circle is d_k. And $k\in[1,(n-1)*6]$

$$d_1=(k-1)\times\sqrt{3}r$$

$$d_2=(n-1)\times\sqrt{3}r$$

$$d_{kt}=\sqrt{d_1^{\,2}+d_2^{\,2}-2d_1d_1\cos(\frac{\pi}{3})}=\sqrt{3}r\sqrt{(n-1)^2+(k-1)^2-(k-1)(n-1)}$$

We use the Sine Theorem in the triangle showed in Figure 10:

$$\frac{d_{kt}}{\sin\frac{\pi}{3}}=\frac{d_1}{\sin\beta}$$

$$\beta=\arcsin\frac{\sqrt{3}d_1}{2d_{kt}}$$

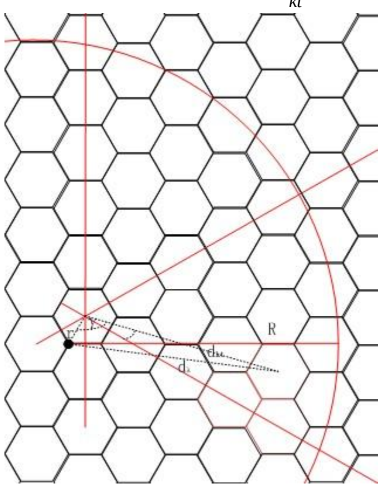


Figure 11 microscopic view

We use the Cosine Theorem in the triangle showed in Figure 11:

$$d_k = \sqrt{d_{kt}^2 + r^2 - 2d_{kt} r \cos\gamma}$$

$$\gamma = \begin{cases} \beta + \frac{5}{6}\pi & \text{section1} \\ \beta + \frac{1}{2}\pi & \text{section2} \end{cases}$$

According to the "Choice Rule", if the $d_k \le R$, the corresponding cell will be concluded in the cellular network, vice verse.

Then we use N_{temp} to count the number of cells from "near layer" and "far layer" which are concluded in the network by the "Choice Rule"

4) Based on the data above, we get the N_p:

$$N_p = 3(N_{near} - 1)^2 + N_{temp}$$

5.6Task Ⅰ

Cellular Communication with Low Population Density in Flat Area

5.6.1 Analysis

Now we have calculated out the crucial parameters in determining the cellular network and made clear about communicating process. To achieve our goal to design an optimal framework for one thousand people communicating simultaneously, we will break down our work into following steps:

- Step One: calculate the population density to check out whether or not one repeater in each cell can meet the demand of users;
- Step Two: design an optimal placement of repeaters using the discrete integer model we build formerly;
- Step Third: distribute the PL tones according to the principle we conclude to make the whole system work successfully.

5.6.2 Solution Details

Step One

We assume that the 1000 users are uniformly distributed. Combined with R_{UtoR} in the flat area(the maximum radius that a repeater can cover, $R_{UtoR} = 8miles$) calculated in 5.3 section, we can easily find that there are only 34 persons in the biggest area a repeater can work. However, the maximum number of users a repeater can serve is 48, which will definitely meet the users' demand. So it is easily concluded that only one repeater is needed in each cell.

Step Two

With the users' density and the crucial parameters, we stimulate the discrete integer model to get the relation of a cell's radius and cell's total number. **Figure 12** demonstrates the result using the strategy one, and **Figure 13** demonstrates the result using the strategy two.

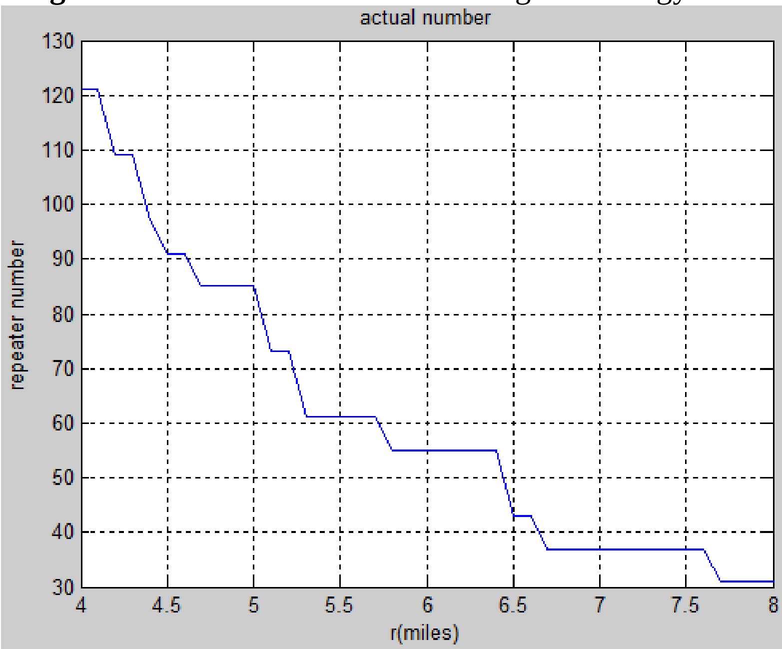


Figure 12 result by strategy one

From the Figure 13, we can see that the relation between the radius of each cell and repeater's total is strictly inversely proportional. So we choose 8 miles as the best radius of each cell. In this case, the minimum number of repeaters is 31, and each cell holds 34 persons.

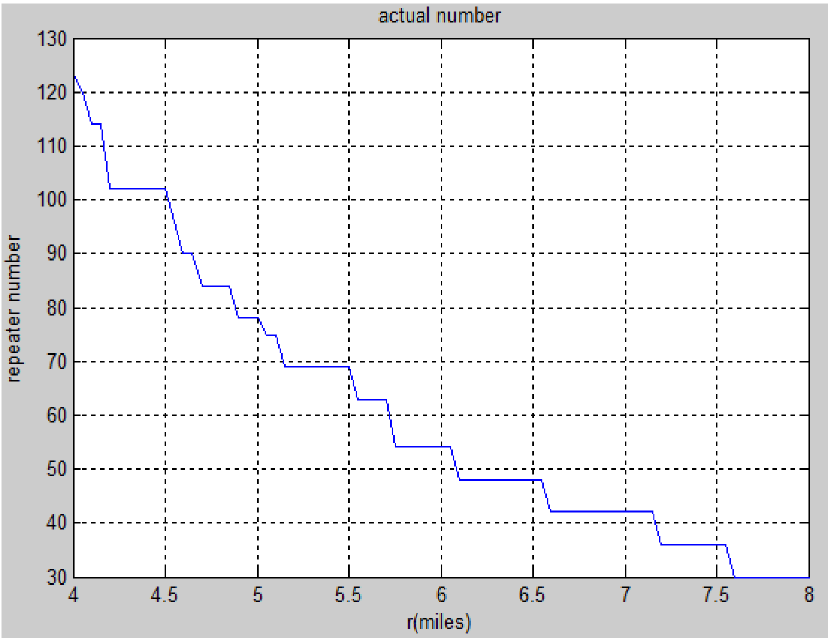


Figure 13 result by strategy two

Figure 14 tells us the result got by strategy two is much similar to the result got by strategy one. 8 miles should still be the best radius of each cell. However, the minimum number got by this strategy decrease to 30. Therefore, we choose the placement by strategy two as our final result.

Step Three

According to the principle of communication we concluded in 5.4 section. Because the total of repeaters is no larger than 54, we could use 30 repeaters which are all from the same group to be placed in the cell. And in terms of the PL tones, the same tones shouldn't be used in the adjacent repeaters. With these constrains, we can frame out the optimal placement of repeaters.

5.6.3 Result

Final result: When the repeater's uplink distance is 8 miles and the users are uniformly distributed, the minimum total of repeaters is 30. And we use 4 kinds of PL to identify them.

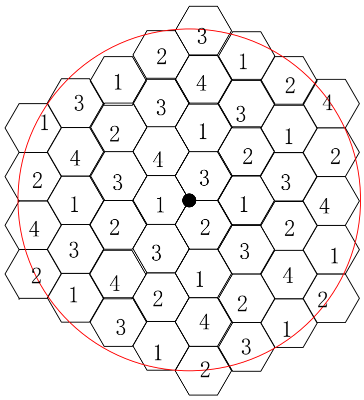


Figure 14 final result

Figure 14 gives us a glimpse of our optimal placement of repeaters. Each cell is placed with a repeater in the center. The number in each cell presents a kind of PL tones. The same PL tone can be redistributed to another repeater when these repeaters' uplink distance doesn't overlap with each other.

5.7 Task Ⅱ

Cellular Communication with High Population Density in Flat Area

5.7.1 Analysis

The task solution in this section shares the same idea with task Ⅰ.We also generally follow the three steps to frame out the optimal placement. However, there are some slight differences because of the increasing population density

.

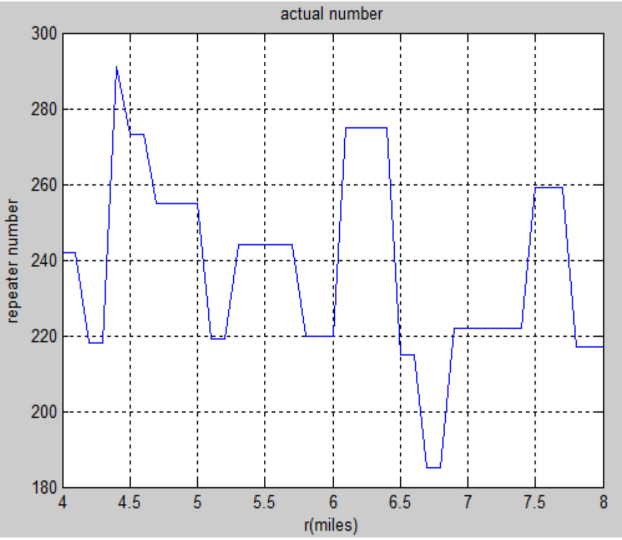
5.7.2 Solution Details

Step One

This task is still based on the assumption that 10000 users are uniformly distributed.And the research area is still focused on flat area, therefore, the R_{UtoR} maintains 8 miles. However, the users' density increases tenfold. So, if the radius of cell is 8 miles, there will be 340 persons in one cell, which one repeater with 48 channels cannot meet the demand. In other words, we need to decrease the radius of cell to make simultaneous communication of 100000 users possible.

Step Two

We stimulate the discrete integer model to find out the best radius of each cell and the minimum total of repeaters to frame out the optimal placement design.**Figure 16** demonstrates the result using the strategy one, and **Figure 17** demonstrates the result using the strategy two.



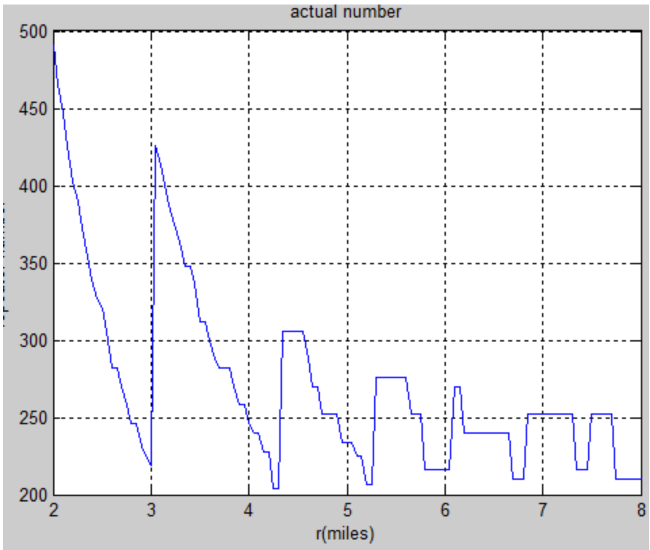


Figure 15 result by strategy one

Figure 16 result by strategy two

Figure 15 shows that as the radius increases, the repeater's total will periodically Jump, and in each period, the relation between the radius of each cell and repeater's total is still inversely proportional. When the radius is 6.8 miles, the repeater's total reaches the minimum. In this case, the minimum number of repeaters is 185.

Figure 16 tells us the result got by strategy two is much similar to the result got by strategy one. 6.8 miles should still be the best radius of each cell. However, the minimum number got by this strategy increase to 210. Therefore, we choose the placement by strategy one as our final result.

Step Three

In these task, five repeaters are placed in one cell. So we should place one group of PL tones to these repeaters to make sure that they can identify themselves from the adjacent repeaters.

5.6.3 Result

Final result: When the radius of each cell is 6.8 miles and the users are uniformly distributed, the minimum total of repeaters is 185. Five repeaters are placed in one cell, and we use 7 group of PL tones to differentiate them.

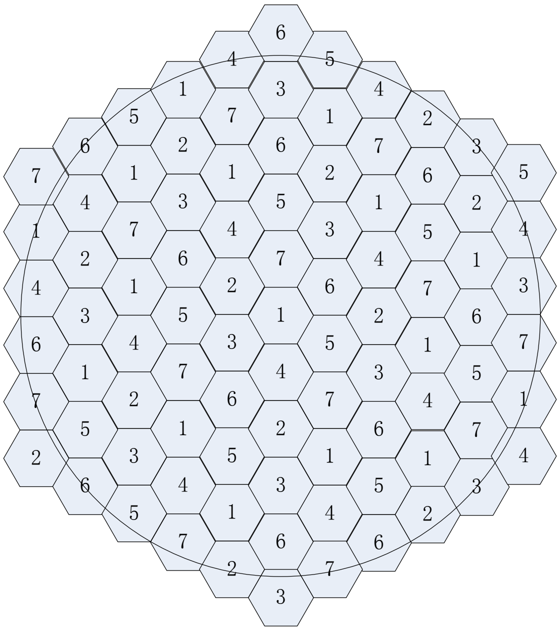


Figure 17 final result

5.8 Task Ⅲ

Cellular Communication in Mountainous Area

5.8.1 Analysis

In mountainous area, the hills will largely impede the signal transmission. So, as we calculated in 5.3 section, the R_{UtoR} decreases to 5 miles. The whole idea of our solution in this situation is much similar to the former two tasks, therefore, through analyzing the Figure 13 to Figure 17, we can get our answer for this task quickly.

From the Figure 13 and Figure 14, because the relation is strictly inversely proportional, we can draw that for serving 1000 users' communication simultaneously, the best radius of the cell is 5 miles. In this case, the minimum total of repeater is 78, and we should choose the strategy two.

From the Figure 16 and Figure 17, we should find the lowest point in the radius's range from one to five miles. Clearly 4.8 miles' radius is preferred, in which case, the minimum total of repeater is 204, and two repeaters are placed in one cell. We choose the strategy two.

5.8.2 Result

When serving for 1000 users simultaneously, the radius of the cell is 5 miles and the minimum of repeaters is 78.

When serving for 10000 users simultaneously, the corresponding radius is 4.8 miles and minimum of repeaters is 204.

Because in the two situations above, we both employ the strategy two as our model, so we get the same form of placement of repeaters. Therefore, we just use **Figure 18** to demonstrate our design.

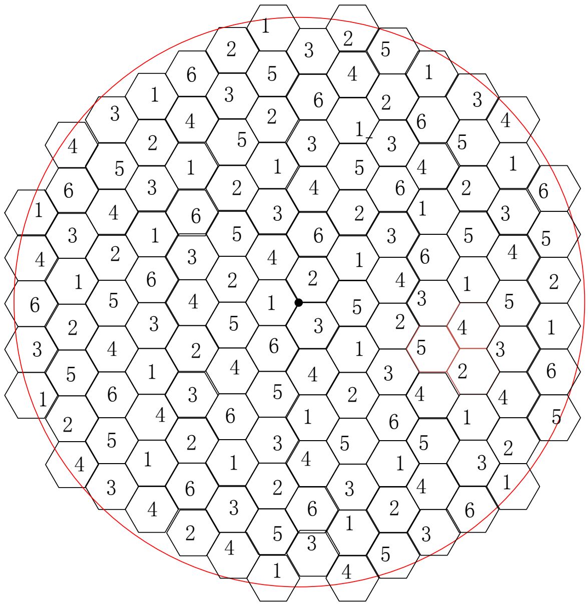


Figure 18 final result

5.9 Validate the Models against a Random Distribution of Population

5.9.1 Introduce

Formerly, our models are built based on the assumption that the people are uniformly distributed. However, in the real-life scenario, people are mobile, so there is no denying that a random distribution of population will suit the reality better. Therefore, to validate our models, we use computer to stimulate a random distribution to analyze their performance.

5.9.2 Symbols Definition

Symbols	Definition
N_p	The practical number of cells in cellular network
P	The number of repeaters in each cell
M_{total}	The total number of people
M_{pi}	The practical number of people in area i
V	The ratio of people who don't get signals
M_{Qosi}	The number of people who don't get communication service in area i
k_{Qos}	A standard for E_{Qos}
k_v	A standard for V
E_{Qos}	The ratio of people who get communication service

5.9.3 Model Details

We bring in two variables---E_{Qos} and V---to help us value the performance of our models. A discrete integer model based on a random population distribution is built as follows:

$$Min \quad N_p \times P$$

$$s.t \quad \begin{cases} \left\lfloor \dfrac{M_p}{C} \right\rfloor & r \le r_{\max} \\ E_{Qos} > k_{Qos} \\ V < k_v \end{cases}$$

V and E_{Qos} is determined as:

$$V = \frac{M_{total} - \sum_i M_{pi}}{M_{total}} \times 100\%$$

$$M_{Qosi} = \begin{cases} 0 & M_{pi} \le PC \\ M_{pi} - PC & M_{pi} > PC \end{cases} \quad i \in (1, N_p)$$

$$E_{Qos} = \left(1 - \frac{\sum_i M_{Qosi}}{M_{total}} \right) \times 100\%$$

We design the following steps to stimulate a random distribution and to find an optimal placement of repeaters at the same time.

- Step One: For a certain radius, find the total of points which is included in the system and locate their coordinate.
- Step Two: Regard the total of repeaters in uniform distribution as the best number for placement.
- Step Three: N points are put into the circle according to the random distribution. Calculate the number of people in every cell to get the in each situation. Then compare them to the k_{Qos} and k_v. If they are good enough, then terminate. If not, go to the Step Four.
- Step Four: Increase the N_p, then go to the Step Three

5.9.4 Conclusion

We assume that the $k_{Qos} = 0.96$, K_v=0.05, and the total number of people is ten thousand. After stimulating the models, we get the analysis of E_{Qos} and V, which are respectively showed in

Figure 19 and **Figure 20**. What's more, we find the relation between the radius and repeater's total in this case demonstrated in **Figure 21**.

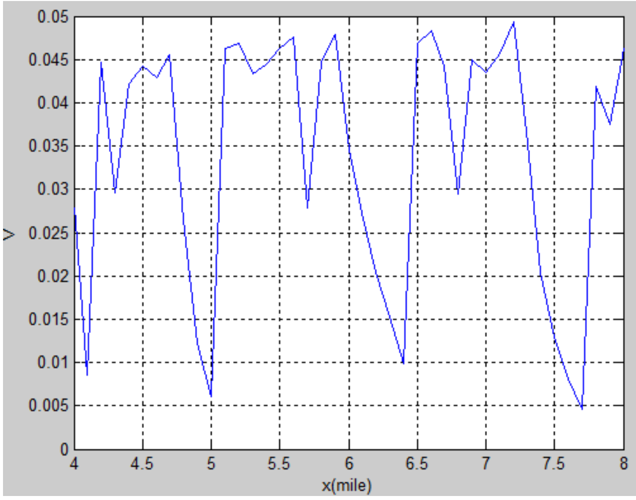


Figure 19 V

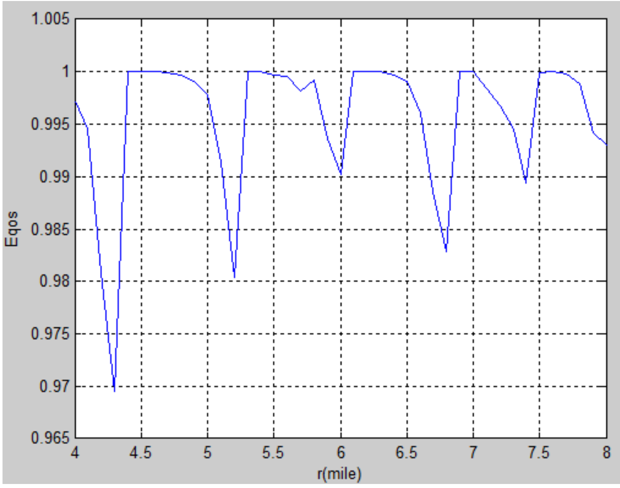


Figure 20E_{Qos}

Through the formula above, we can see that the E_{Qos} and V range from zero to one. And the bigger the E_{Qos} is, smaller the V is, the better the system will be.

The V demonstrated in Figure 19 is extremely small, and the E_{Qos} is almost close to one. Therefore, we can concluded that our models built in Task one to three will suit the random distribution as well. In other words, our models are fully promising using in the real-life scenario.

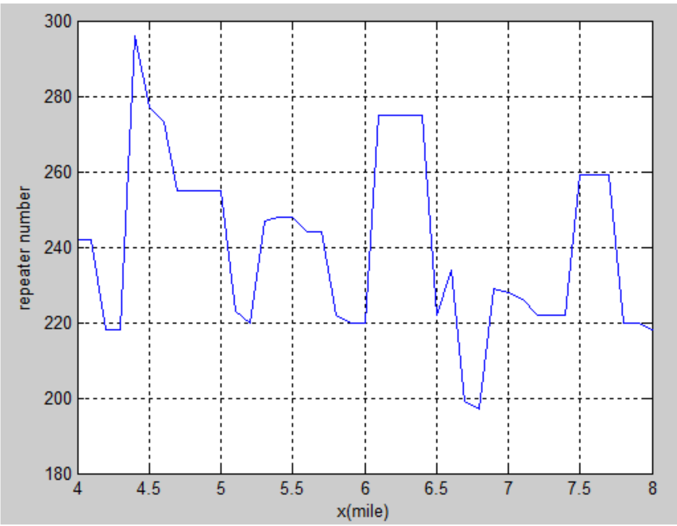


Figure 21

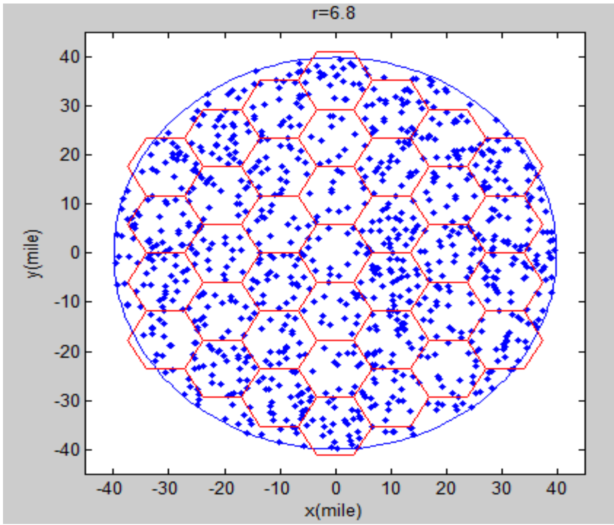


Figure 22

From the Figure 21, we can get that in the case of random distribution, the best radius of the cell is still 6.8 miles and the minimum total of repeaters is 197, which is larger than the previous minimum of 185. The extra can be found in Figure 22.

6.Sensitivity Analysis

In our model, the N_{ch} is a constant. But the N_{ch} has a great impact on the S. We will discuss the influence as follows

$$Y = \frac{\Delta S}{\Delta N_{ch}}$$

S : the optimal repeater number
ΔS : the increment of S
N_{ch} :the max channel for a repeater
ΔN_{ch} : the increment of N_{ch}

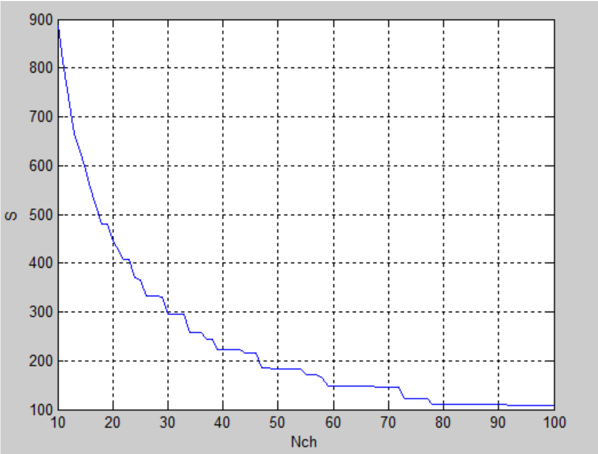


Figure 23

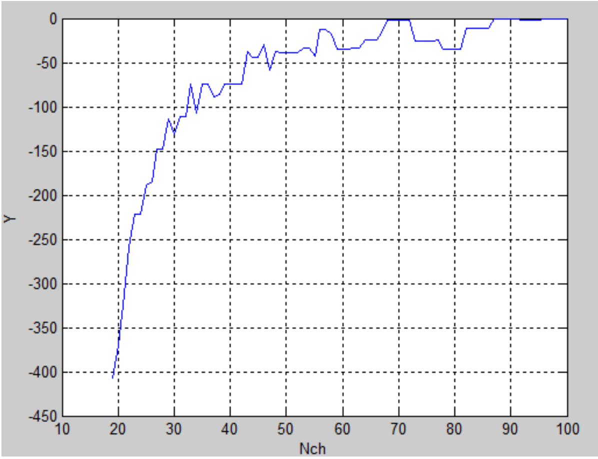


Figure 24

Figure 23 represents the optimal number change with the P. From this figure we can see when P is small the change is quick. While when P increases, the change rate slows down,

Figure 24 represents the change rate which is said above

7. Strengths and Weaknesses

7.1 Strengths

- We use the cellar network coverage model. This model let the area of utilization of a single repeater is large than others. The cellular network model is widely used in communication system, and it is very typical in our life.
- The use of the computer simulation not only let our model easy to implement, but also simplify the calculation. Thus, bring a great convenience into our model solving.

- To control the quality of communication, we define two communication optimization model to evaluate the factors, so that the communication can be maintained at a certain level which we want.
- To count the number of the cells in a system has two strategies. We select the better one for each calculating

7.2 Weaknesses

- The communication mode used in our model is relatively simple. It may fail to satisfy the requirement of a complex communication.
- Consideration for the mountainous model is not perfect. The factors considered are relatively simple.

8. Future Work

We believe that the model would be more optimized when more factors are considered. So the further work should be concentrated on the influence of more factors, such as how specifically the population distribution and geographic distribution effect the optimal strategy selection, establish an optimal Communication network make communication quality to achieve the best.

The future work for repeater coordination could be a multi-objective programming model which includes both the number of repeaters and the cost of establishing and updating a network.

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